A REDUCED-RANK APPROACH TO SINGLE-CHANNEL NOISE REDUCTION

Wei Zhang[1], *Jingdong Chen*[1], *and Jacob Benesty*[2]

[1]: IAIC Research Center
Northwestern Polytechnical University
127 Youyi West Road
Xi'an, Shaanxi 710072, China

[2]: INRS-EMT
University of Quebec
800 de la Gauchetiere Ouest, Suite 6900
Montreal, QC H5A 1K6, Canada

ABSTRACT

This paper investigates the application of the so-called reduced-rank principle to the problem of single-channel noise reduction. Under the reduced-rank framework, we develop reduced-rank Wiener and tradeoff filters. In comparison to the classical Wiener filter, the reduced-rank Wiener one can provide a degree of freedom to control the compromise between noise reduction and speech distortion. Similarly, the reduced-rank tradeoff filter offers one more degree of flexibility than the traditional tradeoff filter to optimize the noise reduction performance. Experiments are carried out and preliminary results indicate that the reduced-rank optimal filters are more interesting than their conventional counterparts in many aspects.

***Index Terms*—** Reduced-rank, noise reduction, Wiener filter, tradeoff filter.

1. INTRODUCTION

The reduced-rank principle, which can be traced back to Shannon's rate-distortion theory [1], is a fundamental and powerful tool to solve many important signal processing problems such as signal coding, dimensionality reduction, signal approximation, to name a few [2], [3]. The basic idea underlying this principle is to find an optimal filter using only a few dominant desired signal subspaces that can hopefully yield a performance as close as possible to, or sometimes even better than that obtained with the conventional solution. This is particularly useful if the desired signal to be processed does not span the entire space where the signal is observed. Then, the reduced-rank adaptive filter, such as the Wiener one developed by Scharf [3], can be used for better estimation and filtering.

In this paper, we investigate the application of the reduced-rank technique to the problem of single-channel noise reduction. The objective of single-channel noise reduction is to estimate a desired speech signal from its noisy microphone observation. Typically, this estimation process is achieved through filtering [4]–[6], where an optimal filter is designed and applied to the noisy signal vector, thereby obtaining an estimate of the desired clean speech. However, this approach does not fully exploit the self-correlation property of speech signals. It is well known that speech signals exhibit strong correlation between neighboring samples and, because of this self-correlation, the dimension of the clean signal subspace is much smaller than that of the noisy signal space. This property has been used in the subspace based noise reduction techniques where bases of the desired signal and noise subspaces are obtained from the eigenvalue decomposition of the noisy covariance matrix. Then, noise reduction is performed by nulling the noise subspace and cleaning the speech-plus-noise subspace via a reweighted reconstruction [7]–[9]. While it may be interpreted as a particular case of the reduced-rank approach [10], the subspace method generally shows less flexibility in designing the noise reduction filter as compared to the reduced-rank technique.

In this paper, we investigate the general reduced-rank approach and apply it to tackle the problem of noise reduction. Under the reduced-rank framework, we develop reduced-rank Wiener and tradeoff filters. In comparison with the classical Wiener filter, the reduced-rank Wiener one can provide a degree of freedom to control the compromise between noise reduction and speech distortion. Similarly, the reduced-rank tradeoff filter offers one more degree of flexibility than the traditional tradeoff filter to optimize the noise reduction performance. Simulations and experiments are presented to justify the claimed merits of the developed reduced-rank noise reduction filters.

This work is supported by Specialized Research Fund for the Doctoral Program of Higher Education Funding No. 20136102110010.

2. SIGNAL MODEL AND PROBLEM FORMULATION

The noise reduction problem considered in this paper is one of recovering the desired signal (clean speech), $x(k)$, k being the discrete-time index, of zero mean from the noisy observation (microphone signal) [5], [6]:

$$y(k) = x(k) + v(k), \tag{1}$$

where $v(k)$ is the unwanted additive noise, which is assumed to be a zero-mean random process (white or colored) uncorrelated with $x(k)$.

In a vector/matrix form, the signal model (1) can be rewritten as

$$\mathbf{y}(k) = \mathbf{x}(k) + \mathbf{v}(k), \tag{2}$$

where

$$\mathbf{y}(k) \triangleq \begin{bmatrix} y(k) & y(k-1) & \cdots & y(k-L+1) \end{bmatrix}^T \tag{3}$$

is a vector of length L, $[\cdot]^T$ denotes the transpose of a vector or a matrix, and $\mathbf{x}(k)$ and $\mathbf{v}(k)$ are defined in a similar way to $\mathbf{y}(k)$. Since $\mathbf{x}(k)$ and $\mathbf{v}(k)$ are uncorrelated by assumption, the correlation matrix (of size $L \times L$) of $\mathbf{y}(k)$ is

$$\mathbf{R_y} \triangleq E\left[\mathbf{y}(k)\mathbf{y}^T(k)\right] = \mathbf{R_x} + \mathbf{R_v}, \tag{4}$$

where $\mathbf{R_x} \triangleq E\left[\mathbf{x}(k)\mathbf{x}^T(k)\right]$ and $\mathbf{R_v} \triangleq E\left[\mathbf{v}(k)\mathbf{v}^T(k)\right]$ are the correlation matrices of $\mathbf{x}(k)$ and $\mathbf{v}(k)$, respectively. The matrix $\mathbf{R_v}$ is assumed to be full rank, i.e., equal to L, but $\mathbf{R_x}$ can be either full rank or rank deficient, depending on the length L and the nature of the desired signal.

978-1-4799-6808-4/14/$31.00

Given the above signal model, the objective of noise reduction is to estimate the desired signal vector, $\mathbf{x}(k)$, from $\mathbf{y}(k)$. This can be achieved by applying a linear transformation to $\mathbf{y}(k)$, i.e.,

$$\mathbf{z}(k) = \mathbf{H}\mathbf{y}(k) = \mathbf{x}_{\rm fd}(k) + \mathbf{v}_{\rm rn}(k), \quad (5)$$

where the vector $\mathbf{z}(k)$ of length L is supposed to be the estimate of $\mathbf{x}(k)$, $\mathbf{H}$ is a square filtering matrix of size $L \times L$, and $\mathbf{x}_{\rm fd}(k) \triangleq \mathbf{H}\mathbf{x}(k)$ and $\mathbf{v}_{\rm rn}(k) \triangleq \mathbf{H}\mathbf{v}(k)$ are the filtered desired speech and residual noise, respectively. Now, the problem of noise reduction becomes one of finding an optimal filtering matrix $\mathbf{H}$ to make the level of $\mathbf{v}_{\rm rn}(k)$ as small as possible so the processed signal is perceived less noisy and, meanwhile, keep $\mathbf{x}_{\rm fd}(k)$ close to $\mathbf{x}(k)$ such that the listener does not perceive much distortion to the desired signal.

3. OPTIMAL CLASSICAL AND REDUCED-RANK FILTERS

We define the error signal between the estimated and desired signals at time k as

$$\mathbf{e}(k) \triangleq \mathbf{z}(k) - \mathbf{x}(k) = \mathbf{H}\mathbf{y}(k) - \mathbf{x}(k). \quad (6)$$

Substituting (2) into (6), we can rewrite the error signal as

$$\mathbf{e}(k) = \mathbf{e}_{\rm sd}(k) + \mathbf{e}_{\rm rn}(k),$$

where

$$\mathbf{e}_{\rm sd}(k) \triangleq \mathbf{x}_{\rm fd}(k) - \mathbf{x}(k) = (\mathbf{H} - \mathbf{I})\,\mathbf{x}(k) \quad (7)$$

is the signal distortion due to filtering, with $\mathbf{I}$ being the $L \times L$ identity matrix, and

$$\mathbf{e}_{\rm rn}(k) \triangleq \mathbf{H}\mathbf{v}(k) \quad (8)$$

represents the residual noise. Now, we can write the mean-square-error (MSE) criterion:

$$\begin{aligned} J(\mathbf{H}) &\triangleq \mathrm{tr}\left\{E\left[\mathbf{e}(k)\mathbf{e}^T(k)\right]\right\} \quad (9)\\ &= \mathrm{tr}\left(\mathbf{H}\mathbf{R}_{\mathbf{y}}\mathbf{H}^T\right) + \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}}\right) - 2\mathrm{tr}\left(\mathbf{H}\mathbf{R}_{\mathbf{x}}\right), \end{aligned}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace of a square matrix.

Since $x(k)$ and $v(k)$ are uncorrelated, it is easy to check that that $E\left[\mathbf{e}_{\rm sd}(k)\mathbf{e}_{\rm rn}^T(k)\right] = \mathbf{0}$. We can then express $J(\mathbf{H})$ as the sum of two other MSEs, i.e.,

$$J(\mathbf{H}) = J_{\rm sd}(\mathbf{H}) + J_{\rm rn}(\mathbf{H}), \quad (10)$$

where

$$J_{\rm sd}(\mathbf{H}) \triangleq \mathrm{tr}\left\{E\left[\mathbf{e}_{\rm sd}(k)\mathbf{e}_{\rm sd}^T(k)\right]\right\}, \quad (11)$$

$$J_{\rm rn}(\mathbf{H}) \triangleq \mathrm{tr}\left\{E\left[\mathbf{e}_{\rm rn}(k)\mathbf{e}_{\rm rn}^T(k)\right]\right\}. \quad (12)$$

With these MSE criteria, we are now ready to derive different optimal filtering matrices.

3.1. Classical Wiener Filter

The classical Wiener filter is the filter that minimizes the MSE $J(\mathbf{H})$, i.e.,

$$\mathbf{H}_{\rm W} = \arg\min_{\mathbf{H}} J(\mathbf{H}). \quad (13)$$

Differentiating $J(\mathbf{H})$ with respect to $\mathbf{H}$ and then equating the result to $\mathbf{0}$, one can easily deduce that

$$\mathbf{H}_{\rm W} = \mathbf{R}_{\mathbf{x}}\mathbf{R}_{\mathbf{y}}^{-1} = \mathbf{I} - \mathbf{R}_{\mathbf{v}}\mathbf{R}_{\mathbf{y}}^{-1}. \quad (14)$$

With this optimal filter, the estimate of $\mathbf{x}(k)$ is

$$\widehat{\mathbf{x}}_{\rm W}(k) = \mathbf{H}_{\rm W}\mathbf{y}(k). \quad (15)$$

3.2. Reduced-Rank Wiener Filter

The reduced-rank Wiener filter is found by minimizing $J(\mathbf{H})$ subject to the constraint that the rank of the filtering matrix, $\mathbf{H}$, is equal to a pre-specified value r [11]–[14]. Mathematically, this problem is expressed as

$$\min_{\mathbf{H}} J(\mathbf{H}) \text{ subject to } \mathrm{rank}(\mathbf{H}) = r. \quad (16)$$

Let

$$\mathbf{R} = \mathbf{R}_{\mathbf{x}}\mathbf{R}_{\mathbf{y}}^{-1}\mathbf{R}_{\mathbf{x}}. \quad (17)$$

It is easy to check that $\mathbf{R}$ is the correlation matrix of $\widehat{\mathbf{x}}_{\rm W}(k)$. With the use of the eigenvalue decomposition, $\mathbf{R}$ can be diagonalized as

$$\mathbf{Q}^T\mathbf{R}\mathbf{Q} = \mathbf{\Lambda}, \quad (18)$$

where $\mathbf{Q}$ and $\mathbf{\Lambda}$ are orthogonal and diagonal matrices containing the eigenvectors and the eigenvalues of $\mathbf{R}$, respectively. Let

$$\mathbf{Q} = \left[\begin{array}{cc} \mathbf{T}_r & \mathbf{\Xi} \end{array}\right], \quad (19)$$

where $\mathbf{T}_r$ is an $L \times r$ matrix consisting of the eigenvectors corresponding to the r largest eigenvalues of $\mathbf{R}$, and the $L \times (L - r)$ matrix $\mathbf{\Xi}$ contains the eigenvectors corresponding to the rest of the eigenvalues of $\mathbf{R}$. It can be verified that $\mathbf{T}_r\mathbf{T}_r^T$ is an orthogonal projection matrix of rank r. With the above eigenvalue decomposition, we can deduce the solution to the optimization problem in (16) [3], [12]–[14]:

$$\mathbf{H}_{\rm RRW} = \mathbf{T}_r\mathbf{T}_r^T\mathbf{H}_{\rm W}. \quad (20)$$

The corresponding estimate of $\mathbf{x}(k)$ is now

$$\widehat{\mathbf{x}}_{\rm RRW}(k) = \mathbf{H}_{\rm RRW}\mathbf{y}(k). \quad (21)$$

It is easy to check that the reduced-rank Wiener filter degenerates to the classical Wiener filter when $r = L$; from this perspective, the latter can be viewed as a particular case of the former.

3.3. Classical Tradeoff Filter

The classical tradeoff filter is obtained by minimizing $J_{\rm sd}(\mathbf{H})$ with the constraint that the residual noise level is smaller than the original noise level. Mathematically, this is equivalent to

$$\min_{\mathbf{H}} J_{\rm sd}(\mathbf{H}) \text{ subject to } J_{\rm rn}(\mathbf{H}) = \beta\mathrm{tr}(\mathbf{R}_{\mathbf{v}}), \quad (22)$$

where $\beta \in (0, 1)$ to ensure that filtering achieves some degree of noise reduction. Using a Lagrange multiplier, $\mu > 0$, to adjoin the constraint to the cost function and assuming that the matrix $\mathbf{R}_{\mathbf{x}} + \mu\mathbf{R}_{\mathbf{v}}$ is invertible, we deduce that the solution to (22) is

$$\begin{aligned} \mathbf{H}_{{\rm T},\mu} &= \mathbf{R}_{\mathbf{x}}(\mathbf{R}_{\mathbf{x}} + \mu\mathbf{R}_{\mathbf{v}})^{-1} \\ &= (\mathbf{R}_{\mathbf{y}} - \mathbf{R}_{\mathbf{v}})[\mathbf{R}_{\mathbf{y}} + (\mu - 1)\mathbf{R}_{\mathbf{v}}]^{-1}. \quad (23) \end{aligned}$$

By adjusting the value of μ in the above filter, one can make a tradeoff between the amount of noise reduction and the degree of speech distortion. In practice it is difficult to determine the optimal value of μ. So, this parameter is in general chosen in a heuristic way. We then have the following three interesting cases.

- If $\mu = 1$, we have $\mathbf{H}_{\mathrm{T},1} = \mathbf{H}_{\mathrm{W}}$, i.e., the tradeoff filter becomes the Wiener filter in this situation.
- If $\mu > 1$, we get a filtering matrix that can achieve more noise reduction than the Wiener one, but at the expense of more speech distortion.
- If $\mu < 1$, we get a filtering matrix that gives less noise reduction than the Wiener filter, but it also introduces less speech distortion.

The estimate of $\mathbf{x}(k)$ with the classical tradeoff filter is

$$\widehat{\mathbf{x}}_{\mathrm{T},\mu}(k) = \mathbf{H}_{\mathrm{T},\mu}\mathbf{y}(k). \quad (24)$$

3.4. Reduced-Rank Tradeoff Filter

The reduced-rank tradeoff filter can be deduced by minimizing $J_{\mathrm{sd}}(\mathbf{H})$ with two constraints: 1) the residual noise level is smaller than the original noise level, and 2) the rank of the filtering matrix, $\mathbf{H}$, is equal to a pre-specified value r. In a mathematical way, this problem can be expressed as

$$\min_{\mathbf{H}} J_{\mathrm{sd}}(\mathbf{H}) \quad \text{subject to} \quad \begin{cases} J_{\mathrm{rn}}(\mathbf{H}) = \beta \mathrm{tr}(\mathbf{R_v}) \\ \mathrm{rank}(\mathbf{H}) = r \end{cases}, \quad (25)$$

where, again, $\beta \in (0, 1)$ and $1 \le r \le L$. Following the same line of derivation of the reduced-rank Wiener filter, we obtain

$$\mathbf{H}_{\mathrm{RRT},\mu} = \mathbf{T}_r\mathbf{T}_r^T\mathbf{H}_{\mathrm{T},\mu}. \quad (26)$$

In comparison with the classical tradeoff filter given in (23), this filter adds another degree of freedom, i.e., rank r, to control the compromise between noise reduction and speech distortion. The estimate of $\mathbf{x}(k)$ is now

$$\widehat{\mathbf{x}}_{\mathrm{RRT},\mu}(k) = \mathbf{H}_{\mathrm{RRT},\mu}\mathbf{y}(k). \quad (27)$$

4. EXPERIMENTS

In this section, we study the reduced-rank optimal filters through experiments and compare their performance to that of the classical optimal filters.

4.1. Experimental Setup

The clean speech used in experiments is recorded from a female speaker in a quiet office environment. It is sampled at 8 kHz and quantized with 16 bits. The overall length of the signal is 30 s. The noisy speech is obtained by adding white Gaussian noise to the clean speech where the noise signal is properly scaled to control the input SNR level.

The implementation of all the noise reduction filters derived in the previous section requires the estimation of the correlation matrices $\mathbf{R_y}$, $\mathbf{R_x}$, and $\mathbf{R_v}$. Computation of $\mathbf{R_y}$ is relatively easy because the noisy signal vector, $\mathbf{y}(k)$, is accessible, but we need a noise estimator to compute the other two matrices. While it is a very important issue, how to effectively estimate the noise or its statistics is beyond what can be covered in this paper. So we will set aside the noise estimation issue and directly compute all the correlation matrices from the corresponding signals. Specifically, at each time instant k, an estimate of the matrices $\mathbf{R_y}$, $\mathbf{R_x}$, and $\mathbf{R_v}$ are computed using the most recent 320 samples (40-ms long) of the noisy, clean, and noise signals, respectively, with a short-time average.

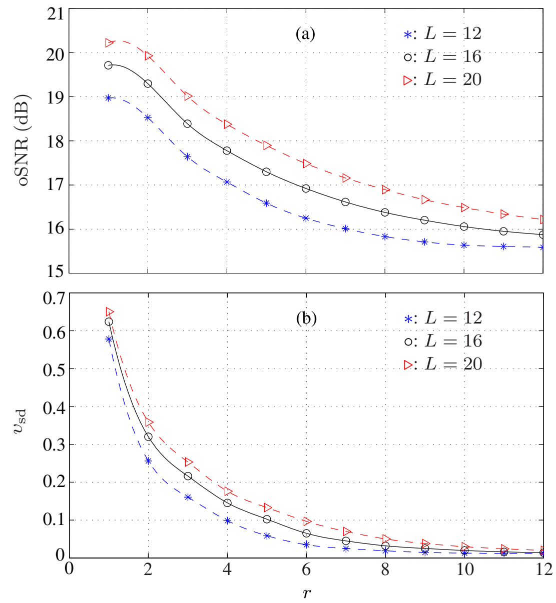

Fig. 1. Performance of the reduced-rank Wiener filter as a function of the rank, r, for three different lengths, L, in white Gaussian noise with 10 dB input SNR: (a) output SNR and (b) speech distortion index.

4.2. Performance Measures

To evaluate the amount of noise reduction, the SNR is adopted as the performance measure. The input SNR, denoted as iSNR, is defined from the signal model given in (2):

$$\mathrm{iSNR} \triangleq \frac{\mathrm{tr}(\mathbf{R_x})}{\mathrm{tr}(\mathbf{R_v})}. \quad (28)$$

The output SNR, obtained from the filter output as given in (5), is

$$\mathrm{oSNR}(\mathbf{H}) \triangleq \frac{\mathrm{tr}\left(\mathbf{H}\mathbf{R_x}\mathbf{H}^T\right)}{\mathrm{tr}\left(\mathbf{H}\mathbf{R_v}\mathbf{H}^T\right)}. \quad (29)$$

To evaluate the amount of speech distortion introduced by the filtering process, we adopt the speech distortion index, which is defined as [6]

$$\upsilon_{\mathrm{sd}}(\mathbf{H}) \triangleq \frac{J_{\mathrm{sd}}(\mathbf{H})}{\mathrm{tr}(\mathbf{R_x})}. \quad (30)$$

Besides the SNR and the speech distortion index, we also compute the perceptual-evaluation-of-speech-quality (PESQ) score, which has been found to have higher correlations, than other widely known objective measures, with the subjective ratings of overall quality of enhanced speech signal [15].

4.3. Performance of the Reduced-Rank Wiener Filter

The first experiment investigates the influence of the length, L, and the rank parameter, r, on the noise reduction performance. The input SNR is set to 10 dB. The results of this experiment are plotted in Fig. 1. It is seen that the output SNR increases with the filter length while the value of the speech distortion index decreases with L. But for L greater than 20, we find that the noise reduction performance does not change much. Note that a larger value of L corresponds to a higher complexity. We, therefore, prefer the filter length as small as possible for reasonably good performance. For a fixed value of L, one can see that both the output SNR and speech distortion index decrease with r. In comparison with the classical Wiener filter, one

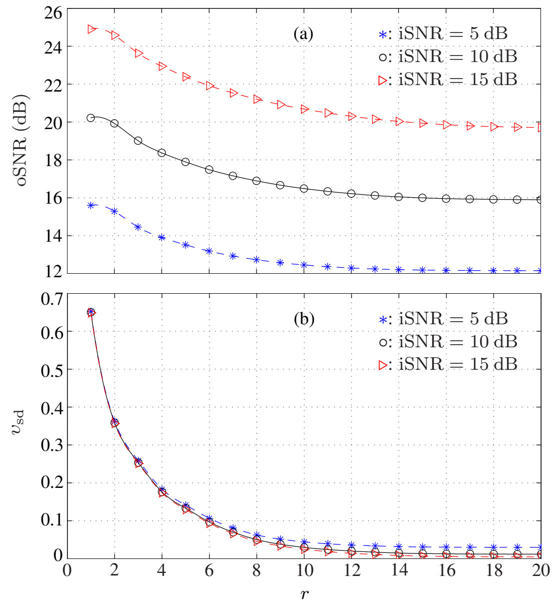

Fig. 2. Performance of the reduced-rank Wiener filter as a function of the rank, r, in white Gaussian noise with three different input SNR levels and $L = 20$: (a) output SNR and (b) speech distortion index.

can see that the reduced-rank Wiener filter provides a nice way to make a compromise between noise reduction and speech distortion, i.e., by choosing a smaller value of rank r, it is possible to achieve a higher output SNR by slightly increasing the speech distortion (if r is not too small) or vice versa.

The second experiment evaluates the performance of the reduced-rank Wiener filter in different SNR conditions. We take $L = 20$. The background noise is again white Gaussian. We test three different SNR conditions, i.e., 5 dB, 10 dB, and 15 dB. The results are plotted in Fig. 2. It is seen that the trend of the output SNR and speech distortion index as a function of the rank parameter r is similar in different SNR conditions, i.e., both decrease with r. But for the three studied cases, we see that the speech distortion index is similar for a given value of r; while we get a higher output SNR if the input SNR is higher. But the SNR gain, which is equal to the output SNR minus the input SNR in decibel scale, is similar in different SNR cases.

4.4. Performance of the Reduced-Rank Tradeoff Filter

In the third experiment, we study the performance of the reduced-rank tradeoff filter as a function of the parameters r and μ. Similar to the previous experiments, white Gaussian noise is used, the input SNR is 10 dB, and $L = 20$. The results for the output SNR and speech distortion index are plotted in Fig. 3 and the PESQ scores are visualized in Fig. 4. It is clearly seen that the reduced-rank tradeoff filter is much more flexible in controlling the compromise between noise reduction and speech distortion than its classical counterpart.

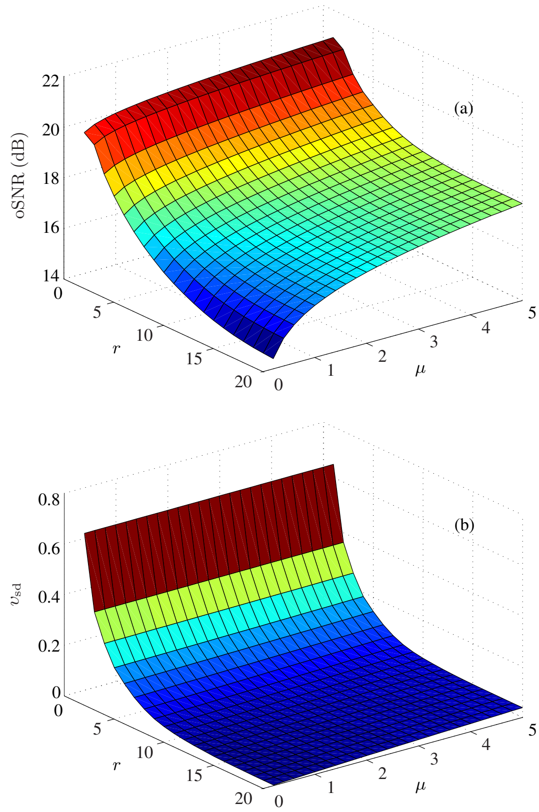

Fig. 3. Performance of the reduced-rank tradeoff filter as a function of the rank parameter, r, and tradeoff parameter, μ, in white Gaussian noise: (a) output SNR and (b) speech distortion index. $\text{iSNR} = 10$ dB and $L = 20$.

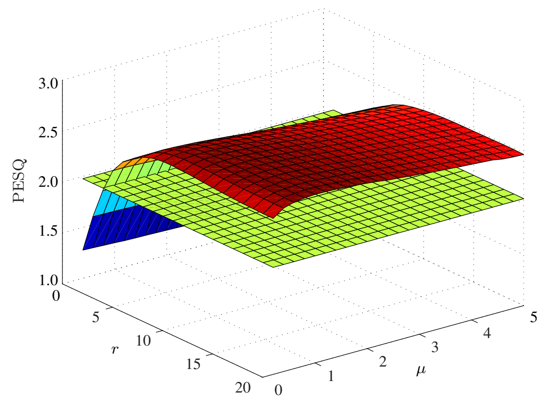

Fig. 4. The PESQ score of the reduced-rank tradeoff filter as a function of the rank parameter, r, and tradeoff parameter, μ, in white Gaussian noise where $\text{iSNR} = 10$ dB and $L = 20$. The green flat surface is the PESQ score between the clean and noisy speech.

5. CONCLUSIONS AND FUTURE WORK

This paper investigated the use of the so-called reduced-rank principle to tackle the problem of noise reduction. Under the reduced-rank framework, we derived reduced-rank Wiener and tradeoff filters. These two filters are found more flexible in controlling the compromise between the amount of noise reduction and the degree of speech distortion than their classical counterparts. Experimental results revealed that the reduced-rank optimal filters has the potential, if the rank parameter is properly chosen, not only to further improve the output SNR but also get almost the same PESQ score of the enhanced speech as compared to their counterparts derived from the classical approach. It remains to be seen, of course, firstly, what other merits the reduced-rank filters can have besides the enhanced performance and tradeoff property and, secondly, how these filters differ from the traditional subspace techniques. Furthermore, it is also of interest to investigate computationally efficient ways to implement those rank-reduced filters.

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