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On the design of differential beamformers with arbitrary planar microphone array geometry

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Abstract: This letter deals with the problem of differential beamforming with microphone arrays of arbitrary planar geometry. By approximating the beampattern with the Jacobi-Anger expansion, it develops an algorithm that can form any specified frequency-invariant beampattern with a microphone array of any planar geometry as long as the sensors' coordinates are given and the spacing between neighboring sensors is smaller than the smallest wavelength. This method is rather general and it can be used to design differential beamformers with linear, circular, and concentric circular differential microphone arrays as well as differential arrays of arbitrary planar geometry where sensors are placed in any specified positions.

[DCS]
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1. Introduction

Differential microphone arrays (DMAs) have now been used in a wide range of applications to enhance broadband acoustic signals while suppressing noise, reverberation, and interference (Bernardini *et al.*, 2017; Byun *et al.*, 2018; Elko, 2004; Sena *et al.*, 2012). Early efforts in this area were focused on linear arrays where differential beamformers are designed in a multistage manner (Elko, 2004; Elko and Meyer, 2008). This method, while elegant and simple, lacks flexibility in controlling white noise amplification, which is a significant problem of DMAs at low frequencies. Recently, an approach was developed to design linear DMAs in the short-time Fourier transform domain with null constraints from the desired beampattern (Benesty and Chen, 2012). It does not only offer the flexibility to design different directivity patterns, but also provides a way to deal with the white noise amplification problem, i.e., improving the white noise gain (WNG) (Elko and Meyer, 2008) by increasing the number of microphones while fixing the DMA order. This approach has been lately extended to the design of circular differential microphone arrays (CDMAs) (Benesty *et al.*, 2015; Huang *et al.*, 2017b), and concentric circular differential microphone arrays (CCDMAs) (Huang *et al.*, 2017a). In comparison with linear DMAs, CDMAs, and CCDMAs enjoy full flexibility in beam steering in a plane.

One can also design three-dimensional (3D) beamformers by using spherical harmonic expansion, which would naturally encompass the two-dimensional (2D) cylindrical expansion (Elko and Meyer, 2008). Generally, the sensor configurations are limited to spherical geometries or circular ones (Yan *et al.*, 2007). In Parra (2006), a method was presented to design frequency-invariant beamformers based on the spherical harmonic decomposition of the beampattern. This method can be adapted to arrays with arbitrary geometry, but then the resulting beampattern is no longer guaranteed to be frequency invariant.

In this paper, we study the more general case of the design of fully steerable 2D DMAs (i.e., can be steered to any angle) with arbitrary planar array geometries where the sensors can be placed in any position as long as their coordinates are accessible to the subsequent beamforming algorithm. This approach can be used to construct any desired directivity pattern and it is rather flexible since the array geometry is no longer restricted to linear and circular ones.

2. Signal model, problem formulation, and performance measures

Let us consider a sensor array consisting of M microphones, which are distributed in a specified area on a plane. Assume that the center of the array coincides with the origin of the 2D Cartesian coordinate system and the azimuthal angles are measured anti-clockwise

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from the x axis. Consider a source signal (plane wave), in the farfield, that propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on the described array. The direction of the source signal to the array is parameterized by the azimuthal angle θ_s. With this model, the steering vector of length M is written as (Monzingo and Miller, 2004),

$$\mathbf{d}(\omega, \theta_s) = \left[e^{\jmath(\omega r_1/c)\cos(\theta_s - \psi_1)} \ \cdots \ e^{\jmath(\omega r_M/c)\cos(\theta_s - \psi_M)} \right]^T, \tag{1}$$

where the superscript T is the transpose operator, $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\omega = 2\pi f$ is the angular frequency, and $f > 0$ is the temporal frequency, r_m ($m = 1, 2,\ldots, M$) is the distance from the mth microphone to the origin point, and ψ_m is the angular position of the mth array element. In the design of DMAs, it is assumed that the spacing between any two neighboring sensors is much smaller than the acoustic wavelength, so that the true acoustic pressure differentials can be approximated by finite differences of the microphones' outputs. In this paper, we consider small-size microphone arrays and assume that this condition easily holds.

The objective of beamforming is to recover the source signal of interest that is corrupted by spatial acoustic noise. For that, the signal received at each microphone is multiplied by a complex weight, $H_m^*(\omega)$, $m = 1, 2, \ldots, M$, where the superscript * stands for complex conjugation. The weighted outputs are then summed together to form the beamformer's output. Stacking all the weights together in a vector of length M, we get

$$\mathbf{h}(\omega) = \left[H_1(\omega) \quad H_2(\omega) \quad \cdots \quad H_M(\omega) \right]^T. \tag{2}$$

Then, the problem of beamforming is to find the optimal filter so that the beamformer's output is a good estimate of the source signal of interest.

Generally, three metrics are used to evaluate the performance of a beamformer, i.e., beampattern, WNG, and directivity factor (DF).

The beampattern describes the sensitivity of the beamformer to a plane wave impinging on the array from the direction θ. Mathematically, it is defined as

$$\mathcal{B}[\mathbf{h}(\omega), \theta] = \mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta) = \sum_{m=1}^{M} H_m^*(\omega) e^{\jmath(\omega r_m/c)\cos(\theta - \psi_m)}, \tag{3}$$

where the superscript H is the conjugate-transpose operator. WNG evaluates the performance of a beamformer with respect to the presence of array imperfection as well as other uncertainties. It is defined as (Elko and Meyer, 2008),

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta_s)|^2}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)}. \tag{4}$$

DF quantifies the ability of the beamformer in suppressing spatial noise from directions other than the look direction. It is written as (Elko and Meyer, 2008),

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{|\mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta_s)|^2}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_d(\omega)\mathbf{h}(\omega)},$$

where $\mathbf{\Gamma}_d(\omega)$ is the pseudo-coherence matrix of the noise signal in a spherically isotropic noise field, and the (i, j)th element of $\mathbf{\Gamma}_d(\omega)$ is $\mathrm{sinc}(\omega\delta_{ij}/c)$, with δ_{ij} being the distance between microphones i and j.

3. Desired beampattern

The main focus of this paper is on the design of DMA patterns. The frequency-independent beampattern of an Nth-order DMA can be written as $\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos(n\theta)$ (Elko, 2004), where $a_{N,n}$, $n = 0, 1,\ldots, N$, are real coefficients determining the shape of the directivity pattern. The coefficients of widely used directivity patterns such as the dipole, the cardioid, the supercardioid, and the hypercardioid, are obtained from different optimal criterion (Sena *et al.*, 2012). For instance, the coefficients of the hypercardioid pattern are obtained from the maximization of the DF and those of the supercardioid pattern are obtained from the maximization of the front-to-back ratio (Elko, 2004; Elko and Meyer, 2008). Given the ideal, target beampattern, the problem of beamforming becomes one of finding the "optimal" beamforming filter, $\mathbf{h}(\omega)$, such that the designed beampattern is as close as possible to the target beampattern. In order to introduce the steering information and connect it to the Jacob-Anger expansion (or circular harmonic expansion), we rewrite the desired frequency-independent beampattern with main beam points in the direction θ_s as

$$\mathcal{B}(\mathbf{b}_{2N}, \theta - \theta_s) = \sum_{n=-N}^{N} b_{2N,n} e^{\jmath n(\theta-\theta_s)} = [\boldsymbol{\Upsilon}(\theta_s)\mathbf{b}_{2N}]^T \mathbf{p}_e(\theta), \tag{5}$$

where

$$\boldsymbol{\Upsilon}(\theta_s) = \mathrm{diag}(e^{\jmath N\theta_s}, \ldots, 1, \ldots, e^{-\jmath N\theta_s}),$$
$$\mathbf{b}_{2N} = \begin{bmatrix} b_{2N,-N} & \cdots & b_{2N,0} & \cdots & b_{2N,N} \end{bmatrix}^T,$$
$$\mathbf{p}_e(\theta) = \begin{bmatrix} e^{-\jmath N\theta} & \cdots & 1 & \cdots & e^{\jmath N\theta} \end{bmatrix}^T.$$

4. Design of differential beamformers with an arbitrary planar geometry

The optimal approximation of the exponential function that appears in beamformer's beampattern, $\mathcal{B}[\mathbf{h}(\omega), \theta]$, from a least-squares error perspective is the Jacobi-Anger expansion (Abramowitz and Stegun, 1964; Huang *et al.*, 2017b), i.e.,

$$e^{\jmath(\omega r_m/c)\cos(\theta-\psi_m)} = \sum_{n=-N}^{N} \jmath^n J_n\left(\frac{\omega r_m}{c}\right) e^{\jmath n(\theta-\psi_m)}, \tag{6}$$

where $J_n(x)$ is the nth-order Bessel function of the first kind with $J_{-n}(x) = (-1)^n J_n(x)$. Substituting Eq. (6) into Eq. (3), we obtain

$$\mathcal{B}_N[\mathbf{h}(\omega), \theta] = \sum_{n=-N}^{N} e^{\jmath n\theta} \jmath^n \boldsymbol{\psi}_n^T(\omega)\mathbf{h}^*(\omega), \tag{7}$$

where

$$\boldsymbol{\psi}_n(\omega) = \left[J_n\left(\frac{\omega r_1}{c}\right) e^{-\jmath n\psi_1} \; J_n\left(\frac{\omega r_2}{c}\right) e^{-\jmath n\psi_2} \cdots \; J_n\left(\frac{\omega r_M}{c}\right) e^{-\jmath n\psi_M} \right]^T, \tag{8}$$

is a vector of length M.

Comparing Eq. (5) with Eq. (7), one can see the following relation:

$$\Psi(\omega)\mathbf{h}(\omega) = \boldsymbol{\Upsilon}^*(\theta_s)\mathbf{b}_{2N}, \tag{9}$$

where

$$\Psi(\omega) = \begin{bmatrix} (-\jmath)^{-N}\boldsymbol{\psi}_{-N}^H(\omega) \\ \vdots \\ \boldsymbol{\psi}_0^H(\omega) \\ \vdots \\ (-\jmath)^N \boldsymbol{\psi}_N^H(\omega) \end{bmatrix}, \tag{10}$$

is a $(2N+1) \times M$ matrix.

Clearly, the design of a directivity pattern of order N requires at least $2N + 1$ microphones. When $M = 2N + 1$, the solution of Eq. (9) is $\mathbf{h}(\omega) = \Psi^{-1}(\omega)\boldsymbol{\Upsilon}^*(\theta_s)\mathbf{b}_{2N}$, which is known to suffer from white noise amplification, particularly at low frequencies. One way to control white noise amplification is by increasing the number of microphones so that $M > 2N + 1$ and then the beamformer is derived by maximizing the WNG. This can be written as the following optimization problem if there is no distortion in the look direction:

$$\min_{\mathbf{h}(\omega)} \mathbf{h}^H(\omega)\mathbf{h}(\omega) \;\; \text{s.t.} \;\; \Psi(\omega)\mathbf{h}(\omega) = \boldsymbol{\Upsilon}^*(\theta_s)\mathbf{b}_{2N}. \tag{11}$$

The solution is

$$\mathbf{h}(\omega) = \Psi^H(\omega)\left[\Psi(\omega)\Psi^H(\omega)\right]^{-1}\boldsymbol{\Upsilon}^*(\theta_s)\mathbf{b}_{2N}. \tag{12}$$

This filter is also the minimum-norm solution of Eq. (9).

With Eq. (12), we can find the differential beamformer that forms the given target beampattern $\mathcal{B}(\mathbf{b}_{2N}, \theta - \theta_s)$ with an arbitrary array geometry (the sensors need to be spaced less than the smallest wavelength). It should be noticed that the developed method can be viewed as a modal matching approach in cylindrical coordinates. The matching accuracy depends on the geometry, but a detailed discussion of this is beyond the scope of this paper.

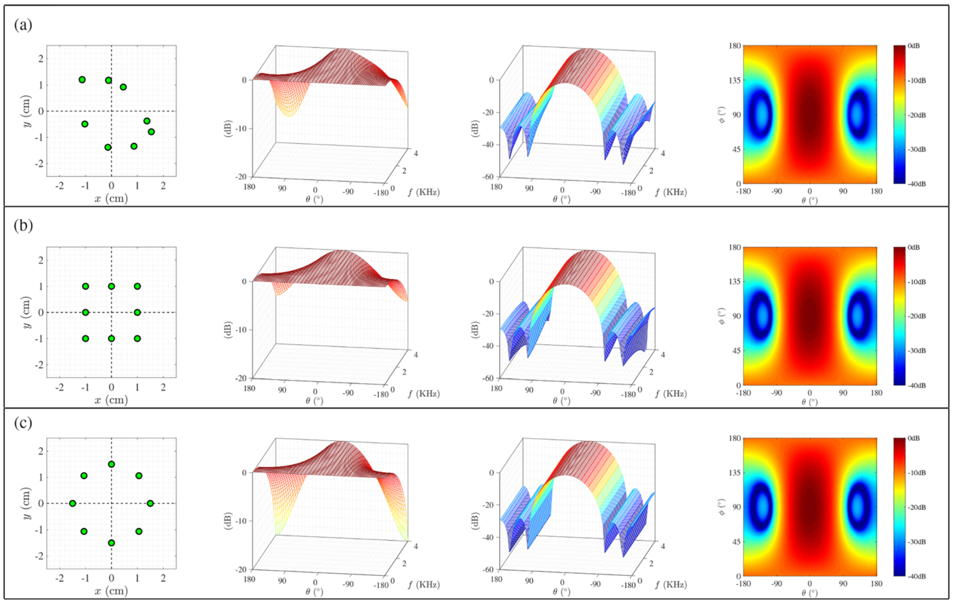

Fig. 1. (Color online) The array geometries, the beampatterns of the corresponding DS beamformer, the 2D and 3D beampatterns of the corresponding differential beamformers designed according to Eq. (12): (a) Array-I, (b) Array-II, and (c) Array-III.

In comparison with our recent works on linear DMAs (Benesty and Chen, 2012) and CDMAs (Benesty *et al.*, 2015; Huang *et al.*, 2017a,b), the major advantage of the developed method is that the array geometry is no longer restricted to linear and circular ones. The method can be used to design DMAs to form a desired directivity pattern with any planar array geometry.

5. Simulations

In this section, we study the performance of the presented method for the design of differential beamformers. The desired (target) beampattern is the second-order hypercardioid, whose coefficients $\mathbf{b}_{2N}$ are obtained from the maximization of the DF (Elko and Meyer, 2008; Benesty and Chen, 2012) and are given by $\mathbf{b}_{2N} = [0.2\ 0.2\ 0.2\ 0.2\ 0.2]^T$. In the simulation, we use eight microphones and consider the following three different array geometries. (1) Array-I: the coordinates of the microphones are random numbers generated with the uniform distribution with the constraints of $1 \leq r_m \leq 2$ cm and $-\pi < \psi_m \leq \pi$. (2) Array-II: a uniform rectangular microphone array of size 2 cm × 2 cm, where the microphones are evenly distributed on four sides. (3) Array-III: a uniform circular microphone array, with a radius of 1.5 cm. Without loss of generality, we assume that the desired look direction is 0°, i.e., $\theta_s = 0°$.

Figure 1 plots the three different geometries, the beampatterns designed with the conventional delay-and-sum (DS) beamformer for comparison, and the corresponding beampatterns designed with the algorithm in Eq. (12). As seen, the DS beamformer has frequency-dependent beampatterns and its directivity is low. The presented method successfully formed the second-order hypercardioid for all three geometries and the designed beampatterns are almost frequency invariant. It is also seen that the beamformer's directivity decreases along the elevation directions off the horizontal plane.

Figure 2 plots the DFs and WNGs of the designed differential beamformers with the aforementioned three array geometries. It is clearly seen that the DF does not

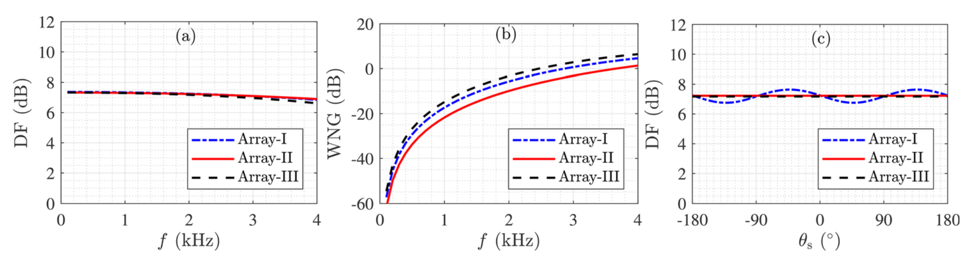

Fig. 2. (Color online) DF and WNG of the differential beamformer designed with the algorithm in Eq. (12) with three different array geometries: (a) DF as a function of the frequency, (b) WNG as a function of the frequency, and (c) DF as a function of the look direction θ_s.

change much with frequency for all three geometries, which corroborates that the designed beampatterns are (almost) frequency invariant.

Figure 2(c) plots the DFs for different look directions, θ_s. It is clearly seen that the designed differential beamformers are continuously steerable. However, note that if all microphones are distributed on a line, full electronic steering is not possible.

6. Conclusions

In this paper, we investigated the problem of differential beamforming with DMAs of arbitrary planar geometries. By approximating the beampattern with the Jacobi-Anger expansion, we developed a beamforming design method, which can approach a specified target frequency-invariant beampattern with an array of any planar geometry as long as the sensors' coordinates are given and the spacing between neighboring sensors is smaller than the smallest wavelength. Simulation results validated the feasibility and effectiveness of the developed differential beamforming algorithm.

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